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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1161**

Unique Paper Code : **237402** **G**

Name of the Paper : **STHT-402; Probability and Statistical Methods-IV**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **IV**

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all, selecting *two* from Section I and *three* from Section II.

Section I

1. (*a*) Let $X_1, X_2, \ldots X_{2n+1}$ be an odd-size random sample from a $N(\mu, \sigma^2)$ population. Find the p.d.f. of the sample median and show that it is symmetric about μ, and hence has the mean μ.

(*b*) Let $X_1, X_2, \ldots, X_n$ be a random sample with common p.d.f :

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the correlation coefficient between $X_{(1)}$ and $X_{(n)}$. 8,7

P.T.O.

2. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$, find the p.d.f., mean and variance of :

$$S = \left(\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2 \right)^{\frac{1}{2}}.$$

(b) For a χ^2 distribution with n d.f. establish the following recurrence relation between the moments :

$$\mu_{r+1} = 2r(\mu_r + n\,\mu_{r-1}),\ r \geq 1.$$

Hence find β_1 & β_2. 8,7

3. (a) Discuss the limiting form of χ^2 distribution for large degrees of freedom.

(b) If X_1 and X_2 are independently distributed each as χ^2 variate with 2 degrees of freedom then show that the probability density function of :

$$Y = \frac{1}{2}(X_1 - X_2)$$

is :

$$g(y) = \frac{1}{2} e^{-|y|},\ -\infty < y < \infty.$$

(c) Show by means of an example that there may exist a random variable X for which E(X) does not exist but $E(X_{(r)})$ exists for some r. 5,5,5

Section II

4. (a) Define the following terms :

(i) Null and alternative hypothesis

(ii) Critical region

(iii) Level of significance.

(b) Define sampling distribution and standard error of a statistic. Obtain the standard error of mean when population size is large.

(c) Let P_1 and P_2 be the (unknown) proportions of students wearing glasses in two universities A and B. To compare P_1 and P_2, samples of sizes n_1 and n_2 are taken from the two populations and the number of students wearing glasses is found to be x_1 and x_2 respectively. Suggest an unbiased estimate of $(P_1 - P_2)$ and obtain its sampling distribution when n_1 and n_2 are large. Hence explain how to test the hypothesis that $P_1 = P_2$. 5,5,5

5. (a) If $\overline{X}$ and S^2 be the usual sample mean and sample variance based on a random sample of n observations

from N(μ, σ²) and $T = (\bar{X} - \mu)\sqrt{n}/S$, where :

$$S^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$$

then prove that :

$$\text{Covariance } (\bar{X}, T) = \{\sigma\sqrt{(n-1)}\,\Gamma(n-2)/2\}/\sqrt{\cdot 2n}\,\Gamma(n-1)/2$$

(*b*) Define Student's t statistic and obtain its p.d.f. Show that Student's t may be regarded as a particular case of Fisher's t statistic. 8,7

6. (*a*) Show that for F-distribution with n_1 and n_2 d.f., the points of inflexion exist if $n_1 > 4$ and are equidistant from the mode.

(*b*) If $X \sim F_{(n, m)}$, then prove that $nX \sim \chi^2$ with n d.f., for large m. 8,7

7. Write short notes on any *three* of the following :

(*i*) Brandt and Snedecor formula for $2 \times k$ contingency table.

(*ii*) *t*-test for difference of means for independent samples.

(*iii*) Applications of F-distribution

(*iv*) Distribution of sample correlation coefficient when population correlation coefficient is zero. 5,5,5